

# Torque Capability Enhanced Method for Five-phase PMSM Drive With Third Harmonic Injection

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**Abstract** — In order to enhance torque density of five-phase permanent magnetic synchronous motor (PMSM) with third harmonic injection, optimum seeking method for injection ratio of third harmonic was proposed adopting theoretical derivation and finite element analysis method, under the constraint of same amplitude of current and air gap flux. The mathematic model which gave the theoretical proof of enhancement effect on torque density by third harmonic injection was deduced. A five-phase PMSM prototype with quasi-trapezoidal flux pattern was designed. Simulation and experimental results prove that using the proposed optimum seeking method, the torque density of five-phase PMSM can be enhanced upto 20%, without any increase of power converter capacity, machine size or iron core saturation.

## I. INTRODUCTION

In the application fields demanding high power grade and high reliability, such as high-power marine, traction, and aerospace applications, the interest in multi-phase machine drive has substantially increased during the last decades because of their enhanced fault tolerance, high power density, high efficiency, high quality torque with lower torque ripple, and the reduction in power ratings for individual power semiconductor devices [1]. Compared to the conventional three-phase counterparts, the multi-phase machines offer additional degrees of freedom essentially which can be used for fault-tolerant operation [2-3], multi-motor series/parallel-connected drive [4] and torque density enhancement [5-7]. The last advantage has shown good prospects for industrial applications. It is based on that the interaction of the spatial and electrical harmonics of the same order can generate a component rotating at fundamental frequency. This component is contributes to positive torque, also providing a flattened MMF shape which is useful to avoid saturation and improve iron utilization.

For five-phase PMSM (FPMSM), this enhancement benefits from the third harmonic airgap flux, which effectively increases the magnitude of the fundamental flux density, without saturating the machine iron, and the third harmonic component also contributes to positive torque meanwhile. The quasi-trapezoidal air-gap flux density due to the combination of the two fluxes is essential for torque density enhancement, assuming the same peak air-gap flux density and phase current amplitude. To get this aim, the stator should be wound such that the induced back EMF is quasi- trapezoidal and is supplied by combined sinusoidal and third harmonic current. So this type of FPMSM is called third harmonic injection FPMSM (THI-FPMSM), which benefits from the controllability of PMSM and high

torque density of BLDC. Little research has been presented relating to determination of windings, permanent magnet shapes and third harmonic current injection ratio.

This paper aims at improving torque density. Theory basis that third harmonic current generates positive and constant torque is given via deduced mathematic model. The design method for a prototype with quasi-rectangular back EMF and optimization method for third harmonic are described. Simulation and experimental results verify the torque density of THI-FPMSM can be enhanced greatly by third harmonic injection.

## II. MODELING OF THI-FPMSM

According to amplitude invariant criterion and extended symmetrical component method, the transformation from natural coordinate system to synchronization rotating coordinate system can be deduced as:

$$T(\theta) = \frac{2}{5} \begin{bmatrix} c(\theta_0) & c(\theta_1) & c(\theta_2) & c(\theta_3) & c(\theta_4) \\ s(\theta_0) & s(\theta_1) & s(\theta_2) & s(\theta_3) & s(\theta_4) \\ c(3\theta_0) & c(3\theta_1) & c(3\theta_2) & c(3\theta_3) & c(3\theta_4) \\ s(3\theta_0) & s(3\theta_1) & s(3\theta_2) & s(3\theta_3) & s(3\theta_4) \\ \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} \end{bmatrix} \quad (1)$$

where  $c(\cdot)$  and  $s(\cdot)$  indicate cosine and sine, respectively.  $\theta_i = -(\theta_r - i\alpha)$ ,  $\theta_r$  is angular displacement of rotor, and  $\alpha = 2\pi/5$ .

Using the transformation of (1), the fundamental and third variables are mapped into two orthogonal subspaces that are referred as  $d_1$ - $q_1$  and  $d_3$ - $q_3$  from now on. So the mathematic model of THI-FPMSM under orthogonal rotating coordinate system is

$$\begin{bmatrix} u_{d1} \\ u_{q1} \\ u_{d3} \\ u_{q3} \end{bmatrix} = r_s \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d3} \\ i_{q3} \end{bmatrix} + \begin{bmatrix} L_{d1} & 0 & L_{13} & 0 \\ 0 & L_{q1} & 0 & L_{13} \\ L_{13} & 0 & L_{d3} & 0 \\ 0 & L_{13} & 0 & L_{q3} \end{bmatrix} \cdot P \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d3} \\ i_{q3} \end{bmatrix} + \omega \begin{bmatrix} -L_{q1}i_{q1} - L_{13}i_{q3} \\ L_{d1}i_{d1} + L_{13}i_{d3} \\ -3L_{13}i_{q1} - 3L_{q3}i_{q3} \\ 3L_{13}i_{d1} + 3L_{d3}i_{d3} \end{bmatrix} + \omega \begin{bmatrix} 0 \\ \psi_{m1} \\ 0 \\ 3\psi_{m3} \end{bmatrix} \quad (2)$$

For  $i_{d1} = i_{d3} = 0$ , the electromagnetic torque can be written as

$$T = \frac{5P}{2} (\psi_{m1}i_{q1} + 3\psi_{m3}i_{q3}) = K_{T1}i_{q1} + K_{T3}i_{q3} \quad (3)$$

where  $P$  denotes the number of pole pairs,  $K_{T1} \equiv 5P\psi_{m1}/2$  and  $K_{T3} \equiv 15P\psi_{m3}/2$  are torque coefficients of fundamental and third harmonic current respectively.

### III. OPTIMUM DESIGN FOR THI-FPMSM DRIVE

#### A. Prototype machine design

This paper designs an eight-pole FPMSM with five identical quasi-concentrated windings as sketched in Fig. 1. The term of quasi-concentrated is used since each phase winding consists of six fractional slot concentrated winding cells, which are in serial connection. Fig. 2 shows the cross section and flux density plots only with the permanent magnet excitation. The magnetic steels of rotor are beveled to inject low-order harmonic and constrain high-order harmonic in the air-gap magnetic field.

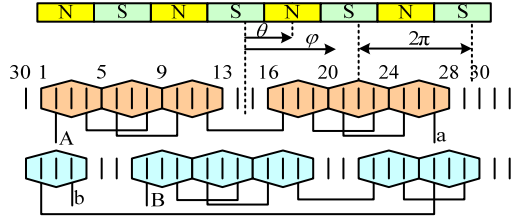


Fig. 1 Winding connections of phase A and B

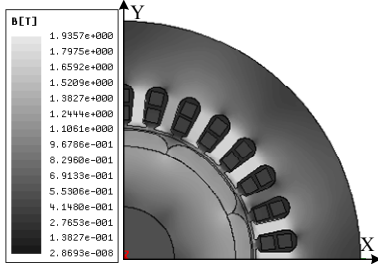


Fig. 2 Cross section and flux density under no load (one quarter)

#### B. Third harmonic current injection ratio optimization

Considering the phase current amplitude is restrained by the power capability of inverter, the third harmonic current injection ratio  $k_3$  must be optimized to maximize the torque density of THI-PMSM, according to the values of  $K_{T1}$  and  $K_{T3}$ . The phase current of phase A can be given as

$$i_{A1} = I_1 \sin(\omega t + \varphi_1) \quad (4)$$

$$i_{A3} = k_3 I_1 \sin(3\omega t + \varphi_3) \quad (5)$$

where  $I_1$  and  $I_3 = k_3 I_1$  are the amplitude values and  $\varphi_1$  and  $\varphi_3$  are the phase angles of the fundamental and third harmonic current components respectively.

Assuming the peak value of phase A is  $I_A = 1$ , (4) and (5) must satisfy:

$$\max(i_{A1} + i_{A3}) \leq I_A \quad (6)$$

According to the theoretical values of  $K_{T1} = 13.7$  and  $K_{T3} = 3.66$ , optimal result can be solved by numerical method. The maximum output torque (16.5746 N·m) will be derived when  $\varphi_1 = \varphi_3$  and  $k_3 = 0.1928$  as shown in Fig. 3.

### IV. EXPERIMENT RESULTS

The measured back-EMF of prototype is illustrated in Fig. 4. The quasi-trapezoidal back-EMF consists of Third and fifth harmonic mainly. And the former is 24.3% of fundamental component, while the latter is eliminated in star-connection.

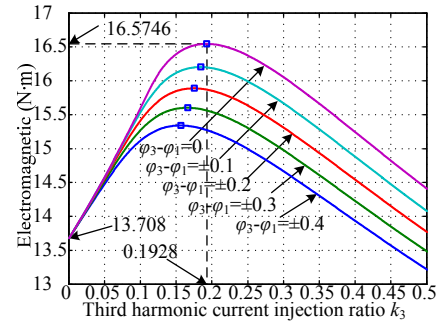


Fig. 3 Optimal design for third harmonic injection ratio

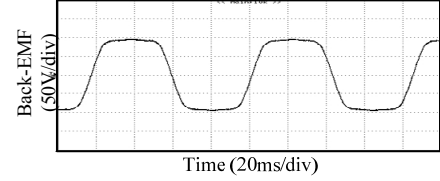


Fig. 4 Measured back-EMF of THI-FPMSM prototype

Fig. 5 shows the currents of phase A and B when  $k_3 = 0$  and  $k_3 = 0.1895$ . Under this two situations, the amplitude of currents is identical, nevertheless the output torque increases from 14.04 N·m to 16.9 N·m, about 20.4%. Obviously, torque density can be enhanced by injecting the third harmonic current by proper ratio and phase without increasing current amplitude.

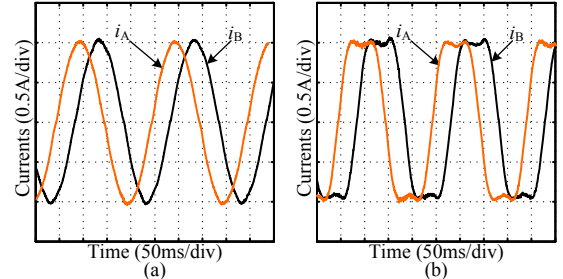


Fig. 5 Currents of phase A and B when (a)  $k_3 = 0$  (b)  $k_3 = 0.1895$

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